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Direct Methods for Finding Optimal Solution of a Transportation Problem are not Always Reliable.

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Abstract: *The transportation model is a special class of the linear programming problem. It deals with the situation in which a commodity is shipped from sources to destinations. The objective is to be determined the amounts shipped from each source to each destination that minimize the total shipping cost while satisfying both the supply limit and the demand requirements. The model assumes that the shipping cost on a given route is directly proportional to the number of units shipped on that route. In general, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling and personnel assignment. So it is very competitive and difficult situation to make a vital decision. In this paper, I have tried to reveal that the proposed direct methods namely Zero-Suffix Method ,SAM-Method for finding optimal solution of a transportation problem do not present optimal solution at all times.*
Key words: *Direct methods, sources, destinations, optimal solution, transportation problem, reliable.*

I. Introduction:

Transportation problems have been widely studied in Computer Science and Operations Research. It is one of the fundamental problems of network flow problem which is usually use to minimize the transportation cost for industries with number of sources and number of destination while satisfying the supply limit and demand requirement. Transportation models play an important role in logistics and supply-chain management for reducing cost and improving service. Some previous studies have devised solution procedure for the transportation problem with precise supply and demand parameters. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. In real world applications, the supply and demand quantities in the transportation problem are sometimes hardly specified precisely because of changing economic conditions.It was first studied by F. L. Hitchcock in 1941, then separately by T. C. Koopmans in 1947, and finally placed in the framework of linear programming and solved by simplex method by G. B. Dantzig in 1951. Since then, improved methods of solutions have been developed and the range of application has been steadily widened. It is now accepted as one of the important analytical and planning tool in business and industry. Several sorts of methods have been established for finding the optimal solution. Among them some methods have been introduced which directly attain at the optimal solution namely Zero Suffix Method, ASM-Method etc. Also it can be said that those methods reveal optimal solution without disturbance of degeneracy condition. There requires least iterations to reach optimality, compared to the existing methods available in the literature. The degeneracy problem is also avoided by those methods. In ASM-Method much easier heuristic approach is proposed for finding an optimal solution directly with lesser number of iterations and very easy computations. But from time to time there occur few evils that, the optimal solution found by them are not actual. In this paper , I have presented that the proposed direct methods for finding optimal solution of a transportation problem do not reflect optimal solution continuously. Three numerical examples are provided to prove my clam. Also by the VAN-MODI process optimal solutions are showed to illustrate the comparison.

Feasible Solution:

A set of non-negative values x_{ij} , I = 1,2,3, ………, and j = 1,2,3,………..,n that satisfies the constraints is called a feasible solution to the transportation problem.

Optimal Solution:

A feasible solution (not necessary basic) is said to be optimal if it **minimizes** the total transportation cost.

Non-degenerate Basic Feasible Solution:

A basic feasible solution to a (m×n) transportation problem that contains exactly m+n-1 allocations in independent positions.

Degenerate Basic Feasible Solution:

A basic feasible solution that contains less that m+n-1 non-negative allocations.

Balanced and Unbalanced Transportation problem:

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in the destinations and is called unbalanced otherwise.
Thus for a balanced problem, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ and
For unbalanced problem, $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

Optimality Test:

Optimality test can be performed if "the number of allocation cells in an initial basic feasible solution = m+n-1(No. of rows + No. of columns - 1)". Otherwise optimality test cannot be performed.
The object of optimality test is that, if we put an allocation in a vacant cell then whether the total transportation cost decreased.
Two methods for optimality test namely "Stepping Stone Method" and "MODI Method" areusually used whereas "MODI Method"is mostly used.

Modified Distribution Method (MODI Method) or u-v Method:

This method follows the following steps:
Step-1: Take the costs only that cells where allocations have. It is called cost matrix for allocated cells..
Step-2: On the above of each column we put v_1 , v_2 , v_3,…….etc and at the same time on the left of each row we put u_1 , u_2 , u_3 , ……etc so that the sum of corresponding u's and v's in every allocated cell is equal to above cost. Then by algebraic calculation, the values of each u's and v's are to be found out.It is called $u_i + v_j$ matrix for allocated cells.
Step-3: The empty cells are filled up by the sum results of corresponding u's and v's.
It is called $u_i + v_j$ matrix for vacant cells..
Step-4: Subtract the above matrix's cells from the corresponding cells of original matrix. It is called cell evaluation matrix.
Step-5: If the above cell evaluation matrix contains only non-negative cells , then the basic feasible solution is optimal.
On the other hand, if the above cell evaluation matrix contains any –ve cell , then the basic feasible solution is not optimal. For optimal solution the following iteration should be run:

Step-1: Select the most negative cell from the above cell evaluation matrix. If there have more than one equal cell, then any one can be chosen.
Step-2: Write the initial basic feasible solution. Give a tick (√) at the most negative entry cell. It is called identified cell.
Step-3: Trace or draw a path in this matrix consisting of a series of alternatively horizontal and vertical lines. The path begins and terminates in the identified cell. All corners of the path lie in the cells for which allocations have been made. The path may skip over any number of occupied or vacant cells.
Step-4: Mark the identified cell as +ve and each occupied cell at the corners of the path alternatively –ve , +ve , -ve and so on.
Step-5: Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a –vesign. Add and subtract this new allocation from the cells at the corners of the path , maintaining the row and column requirements. This causes one basic cell to become zero and other cells remain non-negative. The basic cell whose allocation has been made zero , leaves the solution.

VAM-MODI process:

VAM-MODI process indicates, initial basic feasible solution is calculated by "Vogel's Approximation Method" first then optimality test can be checked out by "Modified Distributive Method".

II. ASM-Method:

Step 1: Construct the transportation table from given transportation problem.
Step 2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.

Step 3: Now there will be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose $(i, j)^{th}$ zero is selected, count the total number of zeros (excluding the selected one) in the i^{th} row and j^{th} column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.
Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a $(k,l)^{th}$ zero breaking tie such that the total sum of all the elements in the k^{th} row and l^{th} column is maximum. Allocate maximum possible amount to that cell.
Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.
Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat step 2, otherwise go to step 7.
Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

III. Zero Suffix Method:

We, now introduce a new method called the zero suffix method for finding an optimal solution to a transportation problem .The zero suffix method proceeds as follows.
Step 1: Construct the transportation table for the given TP
Step 2: Subtract each row entries of the transportation table from the row minimum and then
subtract each column entries of the resulting transportation table after using the Step 1 from the column minimum.
Step 3: In the reduced cost matrix there will be at least one zero in each row and column, then
find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S, Therefore S={Add the costs of nearest adjacent sides of zeros/ No. of costs added}
Step 4: Choose the maximum of S, if it has one maximum value then first supply to that
demand corresponding to the cell. If it has more equal values then select {ai,bj} and supply to that demand maximum possible.
Step 5: After the above step, the exhausted demands (column) or supplies (row) are to be
trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat step2
Step 6: Repeat Step 3 to Step 5 until the optimal cost is obtained.

Numerical Examples:

Problem-1:
Consider the following cost minimizing transportation problem with four sources and three destinations:

		Sources 1	2	3	4	Supply
Destinations	A	13	18	30	8	8
	B	55	20	25	40	10
	C	30	6	50	10	11
Demand		4	7	6	12	

By applying VAM-MODI process allocations are obtained as follows:

					Supply
	13 4	18	30	8 4	8
	55	20 4	25 6	40	10
	30	6 3	50	10 8	11
Demand	4	7	6	12	

The minimum transportation cost associated with this solution is

$$Z = \$\,(13 \times 4 + 8 \times 4 + 20 \times 4 + 25 \times 6 + 6 \times 3 + 10 \times 8)$$
$$= \$\,(52 + 32 + 80 + 150 + 18 + 80)$$
$$= \mathbf{\$412}$$

By applying SAM-Method the allocations are obtained as follows:

					Supply
	13 4	18	30	8 4	8
	55	20 4	25 6	40	10
	30	6 3	50	10 8	11
Demand	4	7	6	12	

The minimum transportation cost associated with this solution is

$$Z = \$\,(13 \times 4 + 8 \times 4 + 20 \times 4 + 25 \times 6 + 6 \times 3 + 10 \times 8)$$
$$= \$\,(52 + 32 + 80 + 150 + 18 + 80)$$
$$= \mathbf{\$412}$$

By applying Zero Suffix Method allocations are obtained as follows:

					Supply
	13 4	18	30	8 4	8
	55	20	25 6	40 4	10
	30	6 7	50	10 4	11
Demand	4	7	6	12	

The minimum transportation cost associated with this solution is

$$Z = \$\,(13 \times 4 + 8 \times 4 + 25 \times 6 + 40 \times 4 + 6 \times 7 + 10 \times 4)$$
$$= \$\,(52 + 32 + 150 + 160 + 42 + 40)$$
$$= \mathbf{\$476}$$

Commits: The VAM-MODI process shows that the optimal solution is \$412 and it is exact and SAM-Method gives the same result. But the Zero Suffix Method gives the wrong result which is not optimal.

Problem-2:
Consider the following cost minimizing transportation problem with six sources and four destinations:

		Sources 1	2	3	4	5	6	**Supply**
	1	9	12	9	6	9	10	5
	2	7	3	7	7	5	5	6
Destinations	3	6	5	9	11	3	11	2
	4	6	8	11	2	2	10	9
Demand		4	4	6	2	4	2	

By applying VAM-MODI process allocations are obtained as follows:

							Supply
	9	12	9 (5)	6	9	10	5
	7	3 (4)	7	7	5	5 (2)	6
	6 (1)	5	9 (1)	11	3	11	2
	6 (3)	8	11	2 (2)	2 (4)	10	9
Demand	4	4	6	2	4	2	

The minimum transportation cost associated with this solution is

$$Z = \$\ (9\times 5 + 3\times 4 + 5\times 2 + 6\times 1 + 9\times 1 + 6\times 3 + 2\times 2 + 2\times 4)$$
$$= \$\ (45 + 12 + 10 + 6 + 9 + 18 + 4 + 8)$$
$$= \mathbf{\$112}$$

By applying SAM-Method the allocations are obtained as follows:

							Supply
	9	12	9 (5)	6	9	10	5
	7	3 (4)	7	7	5	5 (2)	6
	6 (2)	5	9	11	3	11	2
	6 (2)	8	11 (1)	2 (2)	2 (4)	10	9
Demand	4	4	6	2	4	2	

The minimum transportation cost associated with this solution is

$$Z = \$\ (9\times 5 + 3\times 4 + 5\times 2 + 6\times 2 + 6\times 2 + 11\times 1 + 2\times 2 + 2\times 4)$$
$$= \$\ (45 + 12 + 10 + 12 + 12 + 11 + 4 + 8)$$
$$= \mathbf{\$114}$$

By applying Zero Suffix Method allocations are obtained as follows:

							Supply
	9 (4)	12	9 (1)	6	9	10	5
	7	3 (4)	7	7	5	5 (2)	6
	6	5	9	11	3 (2)	11	2
	6	8	11 (5)	2 (2)	2 (2)	10	9
Demand	4	4	6	2	4	2	

The minimum transportation cost associated with this solution is

$$Z = \$ (9 \times 4 + 9 \times 1 + 3 \times 4 + 5 \times 2 + 3 \times 2 + 11 \times 5 + 2 \times 2 + 2 \times 2)$$
$$= \$ (36 + 9 + 12 + 10 + 6 + 55 + 4 + 4)$$
$$= \mathbf{\$136}$$

Commits: The VAM-MODI process shows that the optimal solution is $112 whereas SAM-Method and Zero Suffix Method gives the wrong results which are not optimal.

Problem-3:
Consider the following cost minimizing transportation problem with five sources four destinations:

		Sources 1	2	3	4	5	Supply
	A	4	3	1	2	6	80
Destinations	B	5	2	3	4	5	60
	C	3	5	6	3	2	40
	D	2	4	4	5	3	20
Demand		60	60	30	40	10	

By applying VAM-MODI process allocations are obtained as follows:

	1	2	3	4	5	
A	4 (10)	3	1 (30)	2 (40)	6	80
B	5	2 (60)	3	4	5	60
C	3 (30)	5	6	3	2 (10)	40
D	2 (20)	4	4	5	3	20
	60	60	30	40	10	

The minimum transportation cost associated with this solution is

$$Z = \$(4 \times 10 + 1 \times 30 + 2 \times 40 + 2 \times 60 + 3 \times 30 + 2 \times 10 + 2 \times 20)$$
$$= \$ (40 + 30 + 80 + 120 + 90 + 20 + 40)$$
$$= \mathbf{\$420}$$

By applying SAM-Method the allocations are obtained as follows:

	1	2	3	4	5	
A	4 (10)	3	1 (30)	2 (40)	6	80
B	5	2 (60)	3	4	5	60
C	3 (30)	5	6	3	2 (10)	40
D	2 (20)	4	4	5	3	20
	60	60	30	40	10	

.The minimum transportation cost associated with this solution is

$$Z = \$(4 \times 10 + 1 \times 30 + 2 \times 40 + 2 \times 60 + 3 \times 30 + 2 \times 10 + 2 \times 20)$$
$$= \$ (40 + 30 + 80 + 120 + 90 + 20 + 40)$$
$$= \mathbf{\$420}$$

By applying Zero Suffix Method the allocations are obtained as follows:

	1	2	3	4	5	
A	4 $\boxed{10}$	3	1 $\boxed{30}$	2 $\boxed{40}$	6	80
B	5	2 $\boxed{60}$	3	4	5	60
C	3 $\boxed{30}$	5	6	3	2 $\boxed{10}$	40
D	2 $\boxed{20}$	4	4	5	3	20
	60	60	30	40	10	

The minimum transportation cost associated with this solution is

$$Z = \$(4\times 10 + 1\times 30 + 2\times 40 + 2\times 60 + 3\times 30 + 2\times 10 + 2\times 20)$$
$$= \$\,(40 + 30 + 80 + 120 + 90 + 20 + 40)$$
$$= \mathbf{\$420}$$

Commits: The VAM-MODI process displays that the optimal solution is $420 and at the same time SAM-Method and Zero Suffix Method give the same optimal solution.

IV. Conclusion:

The optimal solution obtained by the SAM-Method and Zero Suffix Method create a haphazard situation and somebody will be confused in decision making whether the solution is optimal or not. So it can not be wise to depend on the optimal solution found by them.

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